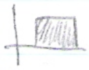
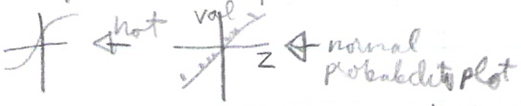
Math 161A Midterm 2 - notes - Kristopher Windsor

$P_{K,n} = \frac{n!}{(n-K)!}$ $\binom{n}{K} = \frac{n!}{K!(n-K)!}$

Uniform dist: 

$\mu_X = E(X) = \int_{-\infty}^{\infty} X f(x)\,dx$ (X or h(x) for some function) in general
$\sigma^2_X = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$



Lognormal $f(x,\mu,\sigma)$ (parameters are of $\ln(X)$)
- pdf $= \frac{e^{-(\ln(x)-\mu)^2/(2\sigma^2)}}{\sigma x \sqrt{2\pi}}$ when $x \geq 0$
- cdf $= \Phi((\ln(x)-\mu)/\sigma)$
- $E(X) = e^{\mu+\sigma^2/2}$, $V(X) = e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$

$\boxed{\int u\,dv = uv - \int v\,du}$ integrate → $\ln(a)a^x$ → a^x → $a^x/\ln(a)$

Beta $f(x,\alpha,\beta,A,B)$
- $\frac{1}{B-A} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{x-A}{B-A}\right)^{\alpha-1} \left(\frac{B-x}{B-A}\right)^{\beta-1}$ if $A \leq x \leq B$
- for standard beta, $A=0$, $B=1$
- $\mu = A + (B-A)\alpha/(\alpha+\beta)$, $\sigma^2 = \frac{(B-A)^2\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

MVUE - minimum variance unbiased estimator

$p_{\bar{X}}(5) = P(\bar{X}=5) =$ ways $\bar{X}=5$ / total ways ("sampling distribution")

$V(X) = E(X^2) - E(X)^2$, $E(X) = \sum x\,p(x)$

Distribution of Sample Mean:
- $E(\bar{X}) = E(X)$, $\sigma^2_{\bar{X}} = \sigma^2_X/n$; $\sigma_{\bar{X}} = \sigma_X/\sqrt{n}$
- (for total T), $E(T) = n\mu$, $V(T) = \sigma^2 n$
- for large n, $\bar{x}$ is normally distributed ($n > 30$)

if $Y = X_1 X_2 X_3 \ldots$ for large n, Y is lognormal

Linear Combination $Y = a_1 X_1 + \ldots + a_n X_n$
- $E(Y) = a_1\mu_1 + \ldots + a_n\mu_n$
- $V(Y) = \sum\sum a_i a_j \,Cov(X_i, X_j)$
- if independence: $V(Y) = a_1^2\sigma_1^2 + \ldots + a_n^2\sigma_n^2$

Chi-Squared: $f(x,v) = \text{gamma}(x, v/2, 2)$

Weibull Distribution (generalized exponential)
- $\text{pdf}(x,\alpha,\beta) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha}$, $x \geq 0$
- $\text{cdf}(x,\alpha,\beta) = 1 - e^{-(x/\beta)^\alpha}$
- $\mu = \beta\Gamma(1+1/\alpha)$, $\sigma^2 = \beta^2(\Gamma(1+\frac{2}{\alpha}) - \Gamma(1+\frac{1}{\alpha})^2)$

central limit theorem → $\bar{x}$ is normal for large n

point estimate has value E(X) and error σ
bias $= E(\hat{\theta}) - \theta$
IE for binomials, $\hat{p} = X/n \approx p$

$\hat{\sigma}^2 = \frac{\sum(X-\bar{X})^2}{n-1}$
$\hat{\mu} = \bar{X}$

$\hat{p}$ = estimate $\frac{S}{N}$

	sample	pop
mean	$\bar{X}$	μ
median	$\tilde{X}$	$\tilde{\mu}$
variance	S^2	σ^2
s.d.	S	σ

$$\mathrm{cor}(x,y) = E((X-\mu_x)(Y-\mu_y))$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\mu_x)(y-\mu_y)\,p(x,y)$$

Binomial $b(x,n,p)$
- odds of x wins in n tests with p chance of win
- $\binom{n}{x} p^x (1-p)^{n-x}$
- $E(x) = np$, $V(x) = np(1-p)$

Hypergeometric $h(x,n,M,N)$
- odds ~~of~~ selecting x wins in n samples, with a population N containing M wins
- $\binom{M}{x}\binom{N-M}{n-x} / \binom{N}{n}$
- $E(x) = n\left(\frac{M}{N}\right)$, $V(x) = \frac{(N-n)}{(N-1)}\, n\left(\frac{M}{N}\right)\left(1-\frac{M}{N}\right)$

Negative Binomial $nb(x,r,p)$
- odds of x failures before r wins with p chance of win
- $\binom{x+r-1}{r-1} p^r (1-p)^x$
- $E(x) = r(1-p)p$, $V(x) = r(1-p)p^{-2}$

Poisson $p(x,\lambda)$
- odds of x events in a time period with λ expected events
- $e^{-\lambda}\lambda^x / x!$; approximates binomial $b(x,n,p) \to p(x,np)$
- $E(x) = V(x) = \lambda$ ↳ if $n > 50$, $np < 5$

Normal $f(x,\mu,\sigma)$ (mean, s.d. given) ($\mu=0$, $\sigma=1$ for standard normal)
- pdf is $\frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$
- $Z = \frac{x-\mu}{\sigma}$ ... table lookup result $(Z) = \Phi(z)$
- binomial $b(x,n,p) \approx \Phi((x+.5-np)/\sqrt{npq})$ [if $np \geq 10$, $nq \geq 10$]

Exponential $f(x,\lambda)$
- pdf is $\lambda e^{-\lambda x}$ (or 0 if $x<0$)
- odds the time until event is x; expected wait is $1/\lambda$ [memoryless]
- $E(x) = 1/\lambda$, $\sigma^2 = 1/\lambda^2$
- $F(x) = \mathrm{cdf} = 1 - e^{-\lambda x}$ if $x \geq 0$ else 0

$\mathrm{cdf} = F(x,\alpha,\beta) = F(\frac{x}{\beta},\alpha)$ (use tables)

Gamma $\Gamma(\alpha)$ Function
- $\int_0^\infty x^{\alpha-1} e^{-x}\,dx$
- $\Gamma(n) = (n-1)!$, $\Gamma(1/2) = \sqrt{\pi}$

$E(x) = \alpha\beta$

$V(x) = \alpha\beta^2$

Gamma Distribution $f(x,\alpha,\beta)$, generalized exponential dist
- $f(x,\alpha,\beta) = \left(\frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)}\right)$ if $x \geq 0$, else 0
- standard gamma $\Rightarrow \beta = 1$
- exponential: $\alpha = 1$, $\beta = 1/\lambda$